

The Tale of Two Teachers' Use of Prompts in Mathematical Discussions

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Abstract Facilitating mathematical discussions has consistently been identified as beneficial to students' mathematical learning, with teachers' use of questioning a primary identifier of appropriate facilitation. Although many teachers report familiarity with appropriate questioning techniques, we hypothesized that some teachers may not work in contexts where they can implement what they understand as best practices in their classroom. To explore this potential interaction, two primary teachers with similar dispositions towards mathematics pedagogy, but dissimilar institutional obligations were observed over a 10-week period. The types and frequencies of teachers' questioning and their students' responses during whole class mathematical discussions were observed. Despite both teachers holding similar conceptions of and dispositions towards facilitating mathematical discussion, the effectiveness of teachers' various prompts in eliciting students' mathematical descriptions was substantially different. Findings suggest that differences in the respective teachers' institutional obligations may have affected the effectiveness of one teacher's probing questions.

Keywords: Institutional obligation; mathematical discussion; practical rationality; teacher questioning

1. INTRODUCTION

Students who engage more frequently in talking about, explaining, or discussing mathematics tend to have higher mathematics achievement scores than those students who engage in mathematical discussion less frequently [11, 22]. However, such positive effects depend greatly on the teacher and

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Kosko, KW
Gao, Y

school students are enrolled [23]. Therefore, factors that influence the effectiveness of teachers' facilitation of mathematical discussion and talk are of great importance. Various authors have supplied recommendations to improve the quality of whole class discussion for teachers to implement which recommend teachers: establish certain ground rules for conversation [28]; facilitate dialogic discourse [36], and use appropriate questioning strategies [30]. Within mathematics education research in elementary settings, such strategies have been observed to relate with deeper conceptualizations on the part of students [7, 15]. However, the quality of teachers' questioning in mathematical discussions has been found to relate with teachers' level of mathematical knowledge for teaching (MKT) [13], teachers' descriptions of what counts as effective discussion [12, 24], and teacher's dispositions towards supporting student autonomy [21]. Yet, there is also evidence that curricular demands influence how teachers facilitate mathematical discussion through usage of certain questioning types [1].

Mathematics teachers' pedagogical decisions are influenced by individual resources they bring to such scenarios (knowledge, beliefs, experiences, etc.), but obligations to the teaching profession also influence pedagogical decisions [10]. Within the context of facilitating mathematical discussion, and more particularly regarding teachers' use of questioning in such contexts, it is worth considering to what degree certain professional obligations influence individual teachers' decisions. The present study examines this issue in the case of two teachers with relatively higher demonstrated MKT, and generally positive mathematics pedagogy dispositions, but who taught in different districts with different curricular demands placed. Thus, the purpose of this study is to compare the mathematical questioning practices, and students' responses, of two elementary teachers with similar beliefs, knowledge and dispositions but who taught with different curricular expectations and demands.

2. TEACHER QUESTIONING

A primary means for teachers to facilitate mathematical discussion is through the purposeful use of effective questioning practices. Observations of upper elementary teachers in the U.S. has revealed a connection between a press for meaning via questions that solicited explanation and justification with deepening students' mathematical understandings [19]. Supporting these observations, researchers have found that students whose teachers elicited more explanation and justification via questioning had higher mathematics achievement [11]. Such questions which solicit explanation and justification are also referred to as *probing* questions [1]. Although probing questions are

generally encouraged by findings from observational and empirical studies, another form of questioning is much more prevalent. Referred to as *gathering information* questions, such prompts solicit factual/answer-only responses, recalled/memorized procedures, and similarly simplistic mathematical statements [1]. Gathering information prompts are the dominant form of mathematical questioning by teachers in the U.S., and although such prompts can sometimes stimulate discussion, probing questions have been observed as more consistent in eliciting deeper descriptions of mathematics [34].

Although the questioning that facilitates effective mathematical discussion has been well documented in the literature, the effectiveness of such questioning varies significantly across schools and classrooms [23]. Several research efforts have sought to examine the potential causes of such variance. One description of press for meaning in teachers' facilitation stemmed from observations of four teachers [19]. "All four teachers encouraged their students to describe how they solved the problems and circulated in the room during small-group activity to talk to students about their work" [19]. Yet, teachers who pressed students for meaning sought more than mere descriptions of procedures. They pressed students for explanations through argumentation. Observations of New Zealand teachers' facilitating argumentation [16] support the findings of this prior research [19].

Studying teacher and researcher agreement for teaching logs of instruction, researchers have found that many elementary teachers had different interpretations of justification and proof than did researchers [12]. This difference may be due to elementary teachers' access to mathematical language, since they typically do not have strong backgrounds in mathematics [12]. However, researcher examining high school teachers' depictions of instruction for facilitating mathematical argumentation has found that teachers' interpretations of what counts as argumentation typically lacks an inclusion of probing questions or solicitation of mathematical explanation [24]. When examining for relationships with college-level mathematics coursework, teachers with greater exposure to college-level mathematics were less likely to depict probing questions as a means of scaffolding argumentation [24]. Yet, various research has observed relationships with teachers' specialized content knowledge for teaching mathematics [12, 24] suggesting that particular kinds of mathematical knowledge may relate to which questions teachers pose and how they do so.

Much of the research on teachers' mathematical questioning has identified effective questioning practices [7, 11]. To a lesser degree, the literature also includes study of teachers' conceptions of mathematical questions and facilitation of discussion [12, 24] as well as how teacher knowledge influences

Kosko, KW
Gao, Y

such questioning practices [2, 13, 20, 21]. Yet, factors beyond the individual teachers' control may provide an additional reason for why questioning practices vary in their effectiveness. Comparing the questioning practices of teachers at schools with different curriculum, researchers have observed that teachers with more reform-oriented curriculum posed more probing questions and fewer gathering information questions [1]. A similar finding has been observed in the context of questions on written assessments [6]. Specifically, questions drafted by mathematics teachers can be "affected by the requirements of institutions they work" [6] as well as knowledge and experience, but not necessarily by the curriculum that was mandated. Rather, it was how the particular institutions mandated such curriculum that may have affected such assessment questions. Unfortunately, much of the literature examining teachers' use of questioning in mathematical discussions have focused either on the individual-based resources teachers bring to such scenarios and/or their students' interaction in mathematical discourse as a result of such questioning. The theory of practical rationality affords a means of explaining how individual and institutional factors influence teachers' pedagogy of mathematical questioning.

3. PRACTICAL RATIONALITY OF TEACHER QUESTIONING

There is a triadic relationship between different stakeholders of mathematics instruction (the teacher, the student(s), and the mathematical content), which is commonly referred to as the *instructional triangle* [4]. Among other things, the concept of the instructional triangle allows for explaining how teacher-student interactions can often be mediated by the mathematics content at-hand. From a perspective of explaining how mathematical questioning facilitates discussion, the instructional triangle is useful in characterizing both the relationships between a teachers' questioning and students' response to such questioning, as well as how the mathematics at-hand can mediate this student-teacher interaction.

The literature base suggests certain key resources that aid teachers' posing of appropriate questions in mathematical discussions. These include, but are not limited to, supporting students' autonomy and dialogic discourse [25, 26, 28, 28], possessing a higher degree of MKT [2, 8, 20, 21], and, related to MKT, possessing a developed understanding of appropriate questions for discussion [13, 24]. Although such resources are not necessarily predictive of good questioning practices, the preceding paragraphs describe studies that infer strong relationships with such practices. Yet, as has previously been alluded to, there may be other influences beyond the content, teacher, and particular students in a classroom that influence the use and effectiveness of mathematical questioning. Herbst and Chazan expanded the original

conception of the instructional triangle [4] to include the influence of the institutional environment (or instructional situation) as well as the influence of other various systems that influence students and teachers both in and out of the classroom [10]. The rationale for this inclusion is that teaching includes the activity of instruction, but also activities which influence instruction. Some of these influences stem from particular obligations teachers have to the teaching profession. Four such professional obligations described by researchers [9] include: the disciplinary obligation, or obligation to teach mathematics as a valid representation of the discipline; the individual obligation, or obligation to attend to students as individuals with particular needs and ways of being; the interpersonal obligation, or obligation to the various social dynamics that exist in the classroom context; and the institutional obligation, or obligation to “the department (e.g., textbook choices, curriculum coverage), to the school (e.g., calendar, bell schedules), to the district where the school is (e.g., assessment instruments and goals), to professional associations and unions” [9] and so forth. Although teachers are influenced by each professional obligation [37], the present study focuses specifically on the institutional obligation.

Using alternate language, various studies of mathematics instruction lend support to the claim that teachers' decision-making is influenced by the institutional obligation [9]. For example, organizational mandates alter middle school mathematics teachers' intended actions in the classroom [29]. Similarly, researchers have observed teachers who used an approach they believed was pedagogically unsound because of institutional demands from where they were teaching [18]. Focusing on science teachers, researchers have found that early career teachers in schools with more restrictive curriculum were less likely to teach using reform-oriented pedagogy, which included appropriate questioning practices [17]. Particular to mathematics, teachers' questioning patterns may be influenced by curriculum [1]. Unfortunately, little study has examined how teachers' professional obligations to the institution affect their questioning practices. Yet if, as various research suggests, both individual resources and institutional demands affect the quality of teachers' prompts to facilitate class discussions, it is important to understand the nature of this interaction to improve teaching reform. It is with this focus on such interactions that the present study asks the following research question:

How do the mathematical questioning practices of two elementary teachers' with similar conceptions mathematics pedagogy, levels ofMKT, and dispositions towards discourse vary given differing obligations to their institutions?

4. METHODS

4.1 Participants and Context

Data were collected from two primary grade teachers in a Midwestern U.S. state in the 2012 – 2013 academic year. Mary was a grade 3 teacher with a Bachelors and Master's degree in education. Susan was a grade 1 teacher with a Bachelors in psychology and Master's degree in education. Each teacher had taught for three or more years, indicating they were no longer 'novice' teachers [3]. Mary and Susan previously participated as part of a larger sample in a survey-based study on teachers' perceptions, dispositions and knowledge for facilitating mathematical discussions [21]. Mary and Susan were purposefully selected given the similarity of their self-reported perspectives. Each teacher was observed for 10 mathematics lessons. For the present study, we report on data from both teachers' survey responses and classroom observations.

Prior to being asked to participate in the present study, Mary and Susan completed a survey packet that included a measure of their MKT [14], their disposition for supporting student autonomy [31], their disposition for supporting dialogic discourse in mathematics, and several items regarding their perceived frequency of posing certain types of mathematical questions during discussion [36]. Regarding the MKT score, both Mary and Susan had scores representing higher than average ability¹. The statistic is relevant because prior research has previously observed that teachers with higher MKT generally used questioning effectively to facilitate discussion [13]. Regarding dialogic disposition, both Mary and Susan's scores represent positive dispositions towards dialogic discourse. Finally, both teachers were found to have scores representing support for students' autonomy (as opposed to a controlling classroom). Three additional items assessing teachers' perceived frequency of using certain prompts during discussion were also examined: asking students to explain their procedures; to justify their solution strategies; and to provide a rationale following an explanation. Both teachers reported doing so every or almost every discussion for the first two questioning types and about half discussions for the latter questioning type.

¹Note that the LMT measures are designed to assess the effectiveness of teacher education and professional development efforts, but not individual teachers' teaching ability. Our use of MKT scores should be interpreted in concert with these aims. Thus, MKT provides a proxy for the effectiveness of teachers' prior professional development regarding mathematics content. Specific scores for teachers are not provided to protect the nature of the assessment, and the individual teachers. Both teachers' MKT scores were significantly above the average indicator (0.00).

The institutional obligation states that “the teacher is obligated to observe various aspects of the schooling regime. These include attending to school policies, calendars, schedules, examinations, curriculum, extracurricular activities, and so on” [10]. Mary and Susan indicated similar institutional demands in a number of regards. For example, both teachers had similar amounts of time devoted to mathematics instruction (30 to 40 minutes per day) and often negotiated this allotted time with scheduled assemblies and special events in the school calendar. During the timeframe of observations, each teacher had a shortened mathematics lesson due to a scheduled assembly in their school.

Central to the purpose of the present study are the differences in institutional demands each teacher faced. The different contexts which Mary and Susan taught conveyed different institutional obligations, particularly regarding how each school districts' adopted curriculum was to be interpreted into classroom pedagogy, and different expectations due to the grade levels each teacher taught. Prior to observation, the first author met with each teacher to discuss the teachers' participation, assess their background and disposition, school and district context, and so forth. Mary's school district had adopted the *Saxon* mathematics curriculum and had adopted a policy that mandated teachers were to have students complete a curriculum-provided worksheet each day to reinforce mathematics skills as part of the lesson. This mandate applied to all primary grade levels in the district. Mary indicated concern that these curricular expectations negatively affected her ability to foster effective mathematical discussions – particularly regarding time demands that would not always allow for more in depth discussions. Susan's school district had adopted the *Investigations* mathematics curriculum. Whereas Mary had described specific curricular expectations from her school district, Susan stated she was expected to teach the required content, but was allowed a degree of professional discretion on how to do so.

The U.S. state that Mary and Susan taught required a standardized test be taken in mathematics every year beginning in third grade (Mary's grade level). Students' scores in a teacher's classroom was classified as an indicator of whether a teacher was performing well. Therefore, preparing students for the state mandated test was an additional institutional obligation Mary faced that Susan did not.

4.2 Data and Analysis

The present study incorporated a correlational embedded mixed method design. Specifically, correlational embedded designs collect qualitative

Kosko, KW
Gao, Y

data that is used to help explain relationships and trends in correlational or comparison studies [5]. To fulfill the purpose of this study, Mary and Susan's questioning prompts and their students' responses to such prompts were examined and classified with two coding rubrics. The frequencies of each code allow for a non-parametric comparison of different teacher questions and student responses for each class. A supplementary qualitative analysis of how mathematical discussions were carried out in these classrooms was also conducted. The latter analysis provides a means of explaining observed non-parametric relationships between the two teachers, and provide a richer interpretation of such findings.

4.3 Quantitative Data and Analysis

Observational data was video recorded and transcribed. Transcripts were coded alongside viewing the video for teachers' question types and students' response types. To do this, data was parsed into *exchanges*, which included the opening and closing of discourse regarding particular discursive objects. Following is an example excerpt from Mary's fifth observation (teachers' names are emphasized in bold for readability). In this brief example, the beginning and ending statements serve to open and close the discourse for students to contribute to how the mathematics is conveyed for $53 + \square = 100$. Within each exchange, we initially looked for whether teachers incorporated sequences of questions or if their mathematical prompts were isolated.

Mary: Fifty-three plus what equals one-hundred? What do you think now? Steven?

Steven: Fifty-seven?

Mary: You're close. Jenny?

Jenny: Forty-seven.

Mary: Forty seven... A lot of kids are gonna say fifty-seven. You have to keep in mind what we noticed over here. The digits in the ten's place have to add up to make nine

The above example yields two student responses with three different prompts, but only one of these prompts organized the exchange: "fifty-three plus what equals 100?" Therefore, coding of prompts in an exchange focused on the central prompt, or prompt directing the nature of the exchange. To classify these central prompts, we used Boaler and Brodie's classifications for mathematical

prompts (see Table 1 for descriptions of each code and descriptive statistics) [1]. Both authors coded exchanges independently before coming together to compare and reconcile coding. Prior to reconciling codes, a Kappa statistic of .81 was calculated, suggesting strong interrater reliability for the 583 central prompts coded across both classrooms.

Following coding, we categorized students' responses into six primary codes (shown with descriptive statistics in Table 2). Of particular interest for the present study are three variations of mathematical description: minimal, procedural and rationalized. *Minimal descriptions* conveyed some form of relationships, but provided relatively little more in the way of mathematical

Table 1: Descriptive Statistics for Mary and Susan's Questioning Across 10 Observations.

	Mary	Susan
1. Gathering Information <i>Solicits immediate answers or memorized facts or procedures.</i>	89.5% (n = 323)	89.1% (n = 197)
2. Inserting Terminology <i>Solicits appropriate use of mathematical language.</i>	0.8% (n = 3)	2.7% (n = 6)
3. Exploring Mathematical Meanings and/or Relationships <i>Emphasize or target particular mathematical relationships or meanings.</i>	2.5% (n = 9)	0.9% (n = 2)
4. Probing <i>Solicits elaboration and description from students.</i>	5.3% (n = 19)	2.7% (n = 6)
5. Generating Discussion <i>Solicits contributions from various students.</i>	1.4% (n = 5)	3.6% (n = 8)
6. Linking and Applying <i>Used to make connections to the real-world or to other mathematical topics.</i>	0.3% (n = 1)	0.0% (n = 0)
7. Extending Thinking <i>Connects to similar contexts or tasks.</i>	0.0% (n = 0)	0.0% (n = 0)
8. Orienting and Focusing <i>Focuses on particular elements of a problem or task.</i>	0.3% (n = 1)	0.9 % (n = 2)
9. Establishing Context <i>Connects to topics outside of mathematics.</i>	0.0% (n = 0)	0.0% (n = 0)
Total	362	221

Table 2: Descriptive Statistics for Student Responses across All Observations.

	Mary	Susan
1. Answer-Only <i>Providing an answer to a task.</i>	65.8% (n = 302)	57.1% (n = 220)
2. Affirmation <i>Yes or no answers.</i>	27.9% (n = 128)	21.8% (n = 84)
3. Minimal Description <i>Conveyed a math relationship without procedures.</i>	2.6% (n = 12)	9.4% (n = 36)
4. Procedural Description <i>Conveyed a math relationship with/through procedures.</i>	1.5% (n = 7)	4.7% (n = 18)
5. Rationalized Description <i>Conveyed a math relationship through procedures and justification.</i>	1.3% (n = 6)	4.7% (n = 18)
6. Student Solicits Information <i>A new exchange is initiated by a student with a new central prompt.</i>	0.9% (n = 4)	2.3% (n = 9)
Total	459	385

details (e.g., in response to a word problem: “one person sits in the middle, one person sits on the other side, and one person sits on the other.”). *Procedural descriptions* provided descriptions of mathematical procedurals, but did not provide rationales for these procedures (e.g., “The four plus five equals nine, and then three plus seven equals ten”). *Rationalized descriptions* provided some form of justification or rationale either accompanying procedures or as stand-alone statements (e.g., describing parallel lines: “because even though they’re not the same length, they’re still—they can still go on forever and they’re not gonna cross”). A total of 854 student responses nested in 583 exchanges were coded. Student responses were coded independently by both authors and then reconciled. Interrater reliability of this process indicated strong reliability (Kappa = .91).

5. QUALITATIVE DATA AND ANALYSIS

A microethnographic approach was used to examine how the teachers and students interacted during mathematical discussions. Specifically, microethnography involves examining how certain macro-cultural characteristics manifest themselves in the subtleties of face to face interactions [33]. Microethnography examines such interactions sequentially, meaning that actions inform actions which in turn inform other actions, and so on. Spoken and non-spoken actions of individuals in interactions are of interest,

as well as how such actions build off of one another [27]. A central feature of microethnography is the examination of specific moments in a discourse, as situated in the larger discourse at hand. In this particular study, exchanges (identified in the quantitative data and analysis section) were considered specific moments in larger, whole class mathematical discussions. However, these class discussions were also considered in the context of the teacher's classroom in their particular grade level and school environment. Emergent themes from this analysis were used to make inferences regarding results for the quantitative analysis.

6. RESULTS AND FINDINGS

6.1 Quantitative Results

Table 3 provides a contingency table for Susan and Mary's observed central questions in analyzed exchanges, alongside counts which would be expected by chance. Because certain question types occurred infrequently, they were excluded from quantitative analysis to comply with requirements for calculating the Chi-Square statistic [32]. A comparison between frequencies of prompt types found that these differences were independent from chance ($(df = 4) = 10.12, p = .04$). Specifically, Mary was observed to ask probing questions and explore mathematical meaning questions comparatively more often than Susan. However, Susan was observed to ask insert terminology and generate discussion questions comparatively more often than Mary. By comparatively more often, we mean to suggest that the observed frequencies were comparatively larger than expected by chance alone.

Next, student responses between teachers were examined (see Table 4). Student responses in each class were observed to be statistically significant from chance ($(df = 5) = 40.60, p < .001$). Post hoc analysis of Z-scores for observed and expected differences found that students in Susan's class provided more minimal, procedural and rationalized descriptions than would be expected

Table 3: Contingency table for teachers and observed question types.

	Gather Info	Insert Terminology	Explore Math Meaning	Probing	Generate Discussion
Susan	197 <i>197.0</i>	6 <i>3.4</i>	2 <i>4.2</i>	6 <i>9.5</i>	8 <i>4.9</i>
Mary	323 <i>323</i>	3 <i>5.6</i>	9 <i>6.8</i>	19 <i>e</i>	5 <i>8.1</i>

Note: Observed counts are in normal text, expected counts are italic.

Table 4: Contingency table for students' response types by teachers.

	Answer Only	Affirmation	Minimal Description	Procedural Description	Rational Description	Solicits Info
Susan	220 238.1	84 96.7	36 21.9	18 11.4	18 10.9	9 5.9
Mary	302 283.9	128 115.3	12 26.1	7 13.6	6 13.1	4 7.1

Note: Observed counts are in normal text, expected counts are parenthesis.

by chance, while students in Mary's class provided fewer such descriptions than would be expected by chance. No observable differences were found for answer-only, affirmation or student solicits information responses.

Next, we compared particular student response types with each teacher's questioning prompts (see Table 5) in order to better understand the observed prevalence of various mathematical descriptions in Susan's classroom compared to Mary's. The ratio between student descriptive response and each teacher question type is displayed in Table 5 (i.e., of Susan's six probing questions, there were seven descriptive responses). Interestingly, 20.3% of Susan's gathering information prompts elicited some form of descriptive response from her students, whereas only 3.7% of Mary's gathering information prompts. Additionally, all of Susan's probing prompts elicited some form of description from her students (100.0%) whereas only 26.3% of Mary's probing

Table 5: Comparison of Teachers' Prompts with Students' Descriptive Responses.

	Gather Info	Insert Terminology	Explore Math Meaning	Probing	Generate Discussion	
Susan	Minimal	29	3	0	2	2
	Procedural	14	0	0	2	2
	Rational	15	0	0	3	0
	$S_{\text{Response}} : T_{\text{Question}}$	58 : 197	3 : 6	0 : 2	7 : 6	4 : 8
Mary	Minimal	7	1	1	3	0
	Procedural	2	0	3	1	1
	Rational	4	0	0	2	0
	$S_{\text{Response}} : T_{\text{Question}}$	13 : 323	1 : 3	4 : 9	6 : 19	1 : 5

questions did the same. Although Mary asked a higher proportion of probing questions, Susan's probing questions were found to yield a higher proportion of descriptive responses.

7. QUALITATIVE FINDINGS

Both Susan and Mary were observed to solicit and emphasize descriptions from their students, as well as redirecting students to make connections between each other's mathematical comments and strategies. However, particular differences emerged in examining how both teachers facilitated mathematical discussions. Across observations, Susan generally allowed 1-2 more seconds of wait time than Mary. Maintaining 3 to 5 seconds of wait time significantly improves the quality of discussions [35]. Although Mary generally provided 3 to 5 seconds of wait time, her pace of instruction was quicker than Susan's. Another observation was that Susan's class used manipulatives and other mathematical representations much more frequently, and with more time devoted to these representations. Both of these observed general differences appear to be related to Mary's allocation of mathematics lesson time for students to complete the mandated worksheet during each observed lesson. Specifically, handouts associated with the Saxon curriculum were often integrated within instruction and took at least 10 minutes of allocated math time, on average.

A useful comparison of how these handouts affected mathematical discussions lay in a strategy both Mary and Susan used. Both Mary and Susan would often pause dialogic discourse at a key prompt, but rather than having students answer right away, each would have students work in partners or small groups to investigate the question for a period of time. This partners-as-wait-time strategy often generated better descriptions and more accurate responses from students. Susan's use of this strategy often incorporated teacher-created worksheets for students to record thoughts and ideas while they worked with manipulatives related to the task at-hand. Mary also used worksheets, provided by the mandated curriculum, but these often limited mathematical explorations within the small groups to symbolic manipulation. It is important to note that Mary did use manipulatives and dynamic mathematical representations, but such usage was observed less frequently and with less allocated time than in Susan's lessons. In fact, the manner in which Susan and Mary were observed to engage their students in mathematical discussion often appeared quite similar in the level of detail they were able to elicit from students.

During her third observation, Susan was working with students on identifying properties of shapes. Students were discussing an activity where they attempted to fill various shapes on a worksheet with pattern blocks. One student brought up a word they suggested as another name for a hexagon (i.e., polygon).

Kosko, KW
Gao, Y

Susan: Ah. Guess what? You know what? A polygon- poly means many. So a polygon is a many sided shape. So this is a polygon [holds up hexagon pattern block]. This is a polygon [holds up square pattern block].

Teddy: That's a polygon [points at image on worksheet].

Susan: This is a polygon. Awesome. So they're all polygons. Thank you, Teddy. That was pretty cool.

Jane: Circles aren't polygons!

Susan: Hm.

Cid: Circles are not.

Clay: Yeah they are.

Susan: Would circles be a polygon? Cuz they don't really have-

Teddy: Sides!

Alexa: Sides.

Clay: And they don't have any pointies.

Jessica: Like squares.

Preston: NO vertices.

Susan: NO vertices is what Preston said and Alexa said it has no sides at all.

Alexa: A triangle is!

Cid: It says around.

Preston: It actually could have vertices because it would be- because... you could just make vertices on the sides like little vertices on the sides of the um the oval or circle.

Susan continued facilitating the discussion of what can meet the definition of a polygon, and whether or why a circle would meet the definition. A resolution came about when students tried to fill a drawn circle with pattern blocks and conjectured that no matter what shape you chose you would always have gaps between the shape's side and the circle's edge. Important to notice in the excerpt from Susan's class is the number of students involved in the exchange. This is in contrast to similar situations in Mary's class. Susan provided an initial inserting terminology prompt, but otherwise used revoicing and wait time to facilitate student's engagement.

During Mary's seventh observation, students began discussing a division problem ($8 \div 2$). Disagreement arose when Tim described the division problem with partitioning the eight into two fours. However, Caleb related the problem directly to addition and subtraction.

Mary: Okay, so Tim you started with eight erasers. You divided them into groups of two. How many groups did we end up with?

Tim: Four.

Mary: So it's eight divided by two equals four.

Caleb: Oh I see it's like you have two eights. It's like you have [inaudible] then-

Tim: It's like two.

Caleb: So you take one of that two away and then you have four.

Tim: because you get rid of that two.

Mary: Does this make sense? I'm just not following what you're saying.

Tim: What Caleb is saying makes no sense to me.

Mary: Okay well it must make sense to him. So, does this make sense why it's eight divided by two equals four?

Tim: No.

Caleb: You see this eight.

Mary: Yes.

Caleb: This is like two fours. You take away one of these it would be like one four.

Mary: Okay.

Caleb: So, then you're back at four erasers.

The conversation in Mary's class was eventually resolved by modifying the problem. However, what is important to point out in the above exchange is that only two students were involved in the discussion. By contrast, the exchange provided from Susan's class included multiple students in the exchange. The difference in participants and participation was typical when looking across observations of both teachers. Although each included opportunities for disagreement, and discussion of such disagreement, Mary's facilitation included fewer students in such exchanges, and the exchanges often took

Kosko, KW
Gao, Y

less class time. Mary did press both Tim and Caleb for more clarity in their descriptions. This resulted in a description of Caleb's actions that conveyed an understanding related to subtraction, which Mary recognized. Comparing both excerpts, Susan's class spent more time exploring one student's conjecture (i.e., Preston's conjecture about vertices on a circle) than was done in Mary's class (i.e., Caleb's conjecture of $8 \div 2$ as taking away one group of four).

DISCUSSION

Susan and Mary faced very different institutional obligations involving curriculum and testing expectations. However, both teachers were similar in many ways. Mary and Susan both had positive dispositions towards supporting student autonomy, engaging students in dialogic mathematical discourse, and had higher levels of MKT. Although Mary and Susan had similar dispositions and beliefs regarding mathematical discussion, quantitative results from the present study suggest that Susan's prompts were more effective in eliciting descriptive responses from students. Susan elicited 4.5 times more descriptive responses than Mary. Although a relatively higher proportion of Mary's prompts were probing questions than Susan's, Susan had a higher percentage of student descriptive responses to such prompts than Mary (100% versus 26.3%). Qualitative findings strongly suggest that the context in which teachers taught mattered significantly. One primary reason for this difference may be that Susan allocated more mathematics lesson time to specific exchanges within class discussions than did Mary. Mary and Susan had similar lengths of time for their mathematics lessons, but Mary spent as much as a quarter of her mathematics lesson time having her students complete a mandated worksheet from the district's chosen curriculum. Prior to observation, Mary explicitly described the worksheet as something she felt she had to do because the district had told her to, although she was concerned that following this protocol lessened the quality of her class's mathematical discussions. Another key difference in how time was allocated within discursive exchanges was the larger length wait time provided by Susan compared to Mary. Providing an appropriate amount of wait time is a critical component of effective mathematical discussions [35]. Although wait time is often considered an instructional decision made explicitly by the teacher, findings from the present study suggest that a teacher's decision to include wait time may be influenced by contextual factors related to a teacher's institution.

Describing the theory of practical rationality, researchers suggest that enactment of or deviation from particular actions in the milieu of mathematical instruction can be justified via particular norms and obligations [10]. For the

present study, differences in Mary and Susan's obligation towards the institution was observed to have a substantial effect on their practical rationality for questioning in mathematical discussions. This was evident in Mary and Susan's descriptions of their instructional context, and the observed differences in how time was managed in whole class discussions in mathematics lessons. Further, the findings presented here suggest that particular institutional demands may affect not only what teachers do, or do not do, but students' actions as well. Prior research findings have suggested that students in classrooms where descriptive responses are more prevalent also have higher mathematics achievement scores [11]. Therefore, an important implication of the present study is that teachers who are otherwise knowledgeable in effective pedagogy, but perceive they must adhere to certain institutional obligations, may not teach to their potential. Similar observations regarding other professional obligations have been discussed elsewhere [37]. However, the findings from the present study have important implications specifically regarding the current environment of high stakes testing and its influence on teacher accountability. Rather, in many states, teachers like Mary are evaluated using score-based metrics (i.e., value-added measures). Although we stress that the findings from the present study are not generalizable, and need further investigation, we consider the findings to be highly significant with important implications regarding comparisons of teachers.

The findings presented here are useful in continuing the work of understanding teachers' questioning strategies to facilitate mathematical discussion, as well as factors that influence the enactment of such strategies. Further research comparing teachers of similar conceptions for facilitating mathematical discussion under varying sets of institutional obligations is needed to help illuminate how such institutional demands interact with facilitation of mathematical discussions. By further investigating such interactions, we believe the field will be better informed to improve teacher education and professional development efforts.

REFERENCES

- [1] Boaler, J., & Brodie, K. (2004). The importance of depth and breadth in the analyses of teaching: A framework for analyzing teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the 26th meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 773–782). Toronto, Canada: PME-NA.
- [2] Cengiz, N., Kline, K., & Grant, T. J. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education*, *14*, 355-374. <https://doi.org/10.1007/s10857-011-9179-7>

Kosko, KW
Gao, Y

- [3] Clotfelter, C. T., Ladd, H. F., & Vigdor, J. L. (2007). Teacher credentials and student achievement: Longitudinal analysis with student fixed effects. *Economics of Education Review*, **26**(6), 673–682. <https://doi.org/10.1016/j.econedurev.2007.10.002>
 - [4] Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, **25**(2), 119–142. <https://doi.org/10.3102/01623737025002119>
 - [5] Creswell, J. W., & Plan Clark, V. L. (2007). *Designing and conducting mixed methods research*. Thousand Oaks, CA: Sage Publications. <https://doi.org/10.1177/1558689806298644>
 - [6] Delice, A., Aydin, E., & Cevik, K. S. (2013). Mathematics teachers' use of questions: Is there a change of practice after the curriculum change? *Eurasia Journal of Mathematics, Science & Technology Education*, **9**(4), 417–427.
 - [7] Franke, M. L., et al. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, **60**(4), 380–392. <https://doi.org/10.1177/0022487109339906>
 - [8] Heid, M. K., et al. (2014). A processes lens on a beginning teacher's personal and classroom mathematics. In J. J. Lo, K. R. Leatham, & L. R. Van Zoest (Eds.), *Research Trends in Mathematics Teacher Education* (pp. 67–82). New York: Springer International Publishing. https://doi.org/10.1007/978-3-319-02562-9_4
 - [9] Herbst, P., & Chazan, D. (2011). Research on practical rationality: Studying the justification of actions in mathematics teaching. *The Mathematics Enthusiast*, **8**(3), 405–462.
 - [10] Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM*, **44**(5), 601–612. <https://doi.org/10.1007/s11858-012-0438-6>
 - [11] Hiebert, J. & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, **30**(2), 393–425 <https://doi.org/10.3102/00028312030002393>
 - [12] Hill, H. C. (2005). Content across communities: Validating measures of elementary mathematics instruction. *Educational Policy*, **19**, 447–475. <https://doi.org/10.1177/0895904805276142>
 - [13] Hill, H. C., et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, **26**(4), 430–511. <https://doi.org/10.1080/07370000802177235>
 - [14] Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, **105**(1), 11–30. <https://doi.org/10.1086/428763>
 - [15] Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, **35**(2), 81–116. <https://doi.org/10.2307/30034933>
 - [16] Hunter, R. (2007). Can you convince me: Learning to use mathematical argumentation. In J. H. Woo, H. C., Lew, K. S., Park, & D. Y. Seo (Eds.), *Proceedings of the 31st conference of the International Group for the Psychology of Mathematics Education*, Volume 3 (pp. 81-88), Seoul: PME.
-

-
- [17] Ihrig, L. M. (2014). The effects of socialization on beginning science teachers' pedagogical decision making and science instruction (Unpublished doctoral dissertation). Iowa State University, Ames, Iowa.
- [18] Karaağaç, M. K., & Threlfall, J. (2004). The tension between teacher beliefs and teacher practice: The impact of the work setting. In Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (Vol. 3, 137–144). Toronto, Canada: PME-NA.
- [19] Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, **102**(1), 59–80. <https://doi.org/10.1086/499693>
- [20] Kim, H. J. (2011). An exploratory study of teachers' use of mathematical knowledge for teaching to support mathematical argumentation in middle-grades (doctoral dissertation). Austin, TX: The University of Texas at Austin.
- [21] Kosko, K. W. (2016). Primary teachers' choice of probing questions: Effects of MKT and supporting student autonomy. *IEJME – Mathematics Education*, **11**(4), 991–1012.
- [22] Kosko, K. W. (2012). Student enrollment in classes with frequent mathematical discussion and its longitudinal effect on mathematics achievement. *The Mathematics Enthusiast*, *9*(1&2), 111–148.
- [23] Kosko, K. W., & Miyazaki, Y. (2012). The effect of student discussion frequency on fifth-grade students' mathematics achievement in U.S. schools. *The Journal of Experimental Education*, *80*(2), 173–195. <https://doi.org/10.1080/00220973.2011.566588>
- [24] Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom? *Mathematics Education Research Journal*, **26**(3), 459–476. <https://doi.org/10.1007/s13394-013-0116-1>
- [25] Kosko, K. W., & Wilkins, J. L. M. (2015). Does time matter in improving mathematical discussions? The influence of mathematical autonomy. *The Journal of Experimental Education*, **83**(3), 368–385. <https://doi.org/10.1080/00220973.2014.907225>
- [26] Kosko, K. W., & Wilkins, J. L. M. (2012). Students' quality of mathematical discussion and their self-determination in mathematics. *Investigations in Mathematics Learning*, **4**(3), 15–30.
- [27] MacLure, M. (2003). *Discourse in educational and social research*. Philadelphia: Open University Press.
- [28] Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. London: Routledge.
- [29] Newton, H. Z. (2011). Chinese teachers' self-reported beliefs and practices related to majority and minority students in Xinjiang Uygur Autonomous Region (Unpublished doctoral dissertation). University of Washington, Seattle, WA.
- [30] Oliveira, A. W. (2009). "Kindergarten, can I have your eyes and ears?" politeness and teacher directive choices in inquiry-based science classrooms. *Cultural Studies of Science Education*, **4**, 803–846. <https://doi.org/10.1007/s11422-009-9193-6>
- [31] Reeve, J., Bolt, E., & Cai, Y. (1999). Autonomy-supportive teachers: How they teach and motivate students. *Journal of Educational Psychology*, **91**, 537–548. <https://doi.org/10.1037/0022-0663.91.3.537>
-

Kosko, KW
Gao, Y

- [32] Siegel, S., & Castellan, N. J. (1988). *Nonparametric statistics for the behavioral sciences* (2nd Ed.). New York: McGraw-Hill.
- [33] Streeck, J., & Mehus, S. (2005). Microethnography: The study of practices. In K. L. Fitch & R. E. Sanders (Eds.), *Handbook of language and social interaction* (pp. 381–404). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.
- [34] Temple, C., & Doerr, H. M. (2012). Developing fluency in the mathematical register through conversation in a tenth-grade classroom. *Educational Studies in Mathematics*, **81**, 287–306. <https://doi.org/10.1007/s10649-012-9398-6>
-
- [35] Tobin, K. (1986). Effects of teacher wait time on discourse characteristics in mathematics and language arts classes. *American Educational Research Journal*, **23**(2), 191–200. <https://doi.org/10.3102/00028312023002191>
- [36] Truxaw, M. P., Gorgievski, N., & DeFranco, T. C. (2008). Measuring K-8 teachers' perceptions of discourse use in their mathematics classes. *School Science and Mathematics*, **108**(2), 58–70. <https://doi.org/10.1111/j.1949-8594.2008.tb17805.x>
- [37] Webel, C., & Platt, D. (2015). The role of professional obligations in working to change one's teaching practices. *Teaching and Teacher Education*, **47**, 204–217. <https://doi.org/10.1016/j.tate.2015.01.007>
- [38] Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, **27**, 458–477. <https://doi.org/10.2307/749877>