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A Reflective Rubric-Creating Activity that Enhances Teachers' Mathematical Habits of Mind

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ABSTRACT

As schools and teachers in the U.S. fine-tune their implementation of mathematics standards promoting college and career readiness, the number of support resources continues to expand. One resource focus experiencing significant growth involves sample items and tasks asserting alignment with the college and career ready mathematical content and practice standards. Such samples regularly identify both the content standards addressed and the mathematical habits of mind that students have the potential to engage in. Consistently absent are evaluation criteria articulating how engagement and demonstration of associated mathematical practices can be assessed, concurrent with content. The authors discuss the development of rubrics that attempt to faithfully assess the integration of mathematical content and practice standards and highlight the benefits to mathematics teachers, coaches, professional developers, and mathematics teacher educators of engaging in such reflective rubric-creating activities.

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1. Introduction

As schools and teachers in the U.S. refine and enhance implementation of college and career readiness standards (Common Core State Standards for Mathematics, Nebraska's College and Career Ready Standards for Mathematics), the number of resources supporting enactment continues to expand in both breadth and depth. One resource focus experiencing significant growth involves sample items, tasks, and assessments proclaiming alignment with such standards. Summative assessment resources typically provide stakeholders with practice tests, open access sample assessment tasks, or 'released' items (Partnership for the Assessment of Readiness for College and Careers, Smarter Balanced Assessment Consortium, Minnesota Department of Education). In addition, a variety of organizational and individual entities, not to mention the plethora of mathematics textbook publishers, continue to add to the inventory of available items and tasks (Dan Meyer's Three-Act Math Tasks, Illustrative Mathematics, Achieve the Core's Mathematics Tasks and Assessments). Such sample tasks regularly identify both the content standards addressed and the mathematical habits of mind that students have the potential to engage in (Common Core's Standards for Mathematical Practice, Virginia

Department of Education's Mathematical Process Goals for Students). Frequently accompanying these tasks are instructional recommendations, implementation guidelines, or evaluation criteria, such as rubrics or point systems. Unfortunately, with few exceptions (Education Development Center's Implementing the Mathematical Practice Standards; New York City Department of Education's WeTeachNYC Library), such resources fail to explicate how students might actually engage in the indicated mathematical processes and proficiencies or what such engagement might look like. By 'what engagement might look like', the authors mean "to articulate what a student would need to say or write (communicate) to establish her engagement in particular mathematical habits of mind". Finally, such resources fail to make explicit how demonstrating specific mathematical habits of mind, or lack of such demonstration, impacts student assessment. In this report, the authors put forth and discuss the development of three rubrics that attempt to coherently assess the integration of mathematics content and mathematical habits of mind. Rubric development involved a small sample of prospective and practicing secondary school mathematics teachers (teachers of students ages 14-18 years), referred to as 'participating teachers'. Although the rubrics themselves might appear of limited value, due

to their non-generic nature (their connection to specific content), the rubrics should be considered as secondary to their development—a very cognitive process for everyone involved in their construction.

1. Research Objectives

The main objectives of the rubric-creating activity were to provide a space: (1) for participating mathematics teachers to make their conceptions of mathematical habits of mind explicit and objects of thought, and (2) to explore and support accommodations to participating teachers' conceptions of mathematical habits of mind.

2. Standards for Mathematical Practice

As of 2018, 42 of 50 U.S. states, the District of Columbia, four U.S. territories, and the Department of Defense Education Activity employ unaltered or modified versions of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) that incorporate the Standards for Mathematical Practice (frequently referred to as the mathematical practices, SMPs, or MPs). Some U.S. state college and career readiness standards refer to the Common Core mathematical practices as the Process Standards for Mathematics (Indiana Academic Mathematics Standards) or focus on aligned mathematical processes and proficiencies (Mathematics Standards of Learning for Virginia Public Schools).

Table 1: Alphanumeric Identifier and Title of the Standards for Mathematical Practice

Standards for Mathematical Practice	
Identifier	Title
MP1	Make sense of problems and persevere in solving them
MP2	Reason abstractly and quantitatively
MP3	Construct viable arguments and critique the reasoning of others
MP4	Model with mathematics
MP5	Use appropriate tools strategically
MP6	Attend to precision
MP7	Look for and make use of structure
MP8	Look for and express regularity in repeated reasoning

Due to its widespread application in the U.S., the authors focused on the Standards for Mathematical Practice as defined in the Common Core State Standards for Mathematics (frequently referred to as the Common Core or by the acronym

CCSSM). Table 1 displays the alphanumeric identifier (or code) and title for each of the eight mathematical practices. For a comprehensive description of each mathematical practice, refer to CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). According to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 8), the mathematical practices describe “ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years”. As such, the mathematical practices characterize what mathematics teaching and learning should look like (Sztajn, Marrongelle, Smith, & Melton, 2012, p. 5). As described in *Mathematical Education of Teachers II* (Conference Board of the Mathematical Sciences, 2012, p. 19), “To help their students achieve the [Common Core’s] Standards for Mathematical Practice, teachers must not only understand the practices of the discipline, but how these practices can occur in school mathematics and be acquired by students”. Therefore, to authentically implement instruction that provides students with opportunities to engage in and develop increasingly sophisticated mathematical habits of mind requires teachers with robust conceptions of the mathematical practices. Furthermore, faithful assessment of students’ development of grade- or course-appropriate mathematical habits of mind requires that teachers possess expansive and coherent images for what engagement in each mathematical practice or practice combinations ‘looks like’ and how evidence of such engagement can be elicited both verbally and in students’ written work.

3. Instructional Indicators

A sparse number of Common Core and aligned college and career ready mathematics resources provide student and teacher indicators (commonly referred to as ‘look-fors’) or examples of what engagement in individual or combinations of mathematical practices might look like during classroom discourse. Such resources furnish videos, transcripts, or both, of verbal classroom interactions (Inside Mathematics, Implementing the Mathematical Practice Standards Project); thus, providing “video and narrative exemplars of students using the mathematical practices in pursuit of learning key content” (Sztajn, Marrongelle, Smith, & Melton, 2012, p. 6). Through examination of students’ engagements with rich teaching tasks via video or transcripts of classroom interactions, such resources support teachers’ development of viable images for how the mathematical practices might look during classroom interactions and serve as instructional models of effective classroom discourse.

4. Assessment ‘Look-Fors’

Complimenting these verbal ‘look-fors’ are assessment task resources designed to support teachers in evaluating their students’ development of mathematical knowledge, skills, and habits of mind. According to Daro and Burkhardt (2012, p. 21), “Designing rich assessment tasks that allow all students *to show what they know, understand, and can do* across the range of practices and content set out in CCSSM is among the most challenging areas of educational design”. Though challenging, support resources continue to expand the number of assessment and teaching tasks available for discussion and use in K-12 classrooms, university mathematics methods courses, and professional learning settings.

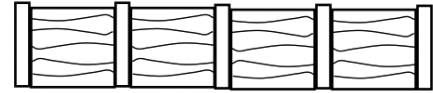
Consistently absent from such resources are accompanying evaluation criteria that make explicit how engagement in the indicated mathematical practices are, or could be, exhibited and assessed concurrent with the content. Specifically, existing evaluation criteria fail to clearly “indicate the qualities by which levels of performance can be differentiated and that anchor judgments about the learner’s degree of success on an assessment” (Stanford Center for Assessment, Learning and Equity, 2013, p. 45)—where levels of performance and degrees of success incorporate both mathematical content and practice standards. For example, the Mathematics Assessment Project’s ‘Fencing’ task illustrated in Figure 1 asks students to “solve real-life and mathematical problems using numerical and algebraic

expressions and equations” (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015, Fencing Task, para. 3).

Fencing

Jon buys fencing for his yard.

He pays \$122 for 5 fence posts and 4 fence panels.



He pays \$570 for 21 fence posts and 20 fence panels.

How much does he pay for 4 fence posts and 3 fence panels? _____

Show how you figured it out.

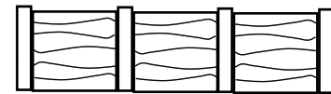


Figure 1: Expressions and equations summative task. This figure illustrates a summative middle school task addressing the seventh-grade expressions and equations domain (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015).

Mathematics Assessment Project documents (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015) provide the Common Core mathematics standards addressed by this item (see Table 2).

Table 2: Alignment for Expressions and Equations Middle School Assessment Task (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015)

Standard(s) for Mathematical Content	Grade	7th (student ages 12-13 years)
	Domain	Expressions and Equations
	Cluster Heading	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
Standard(s) for Mathematical Practice	Standard	MP2 - Reason abstractly and quantitatively MP3 - Construct viable arguments and critique the reasoning of others MP6 - Attend to precision MP7 - Look for and make use of structure

As illustrated in Table 2, the Fencing task, which requires that students determine the cost of building fences using fence posts and wooden panels, is assessed at the cluster level and addresses four mathematical practice standards (MP2, MP3, MP6, and MP7). The Mathematics Assessment Project’s rubric (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015) for evaluating student responses and a sample

of student work are provided in Figure 2. As illustrated in Figure 2, neither the rubric, nor the sample student response provide any indication where students might engage in any of mathematical practices MP2, MP3, MP6, or MP7 (see Table 2). In addition, it does not appear that clear exhibition of engagement in any of these mathematical practices impacts scoring. Finally, it is not clear how the seven points awarded in the sample response align with the rubric (Figure 2).

Fencing	Rubric	
	points	section points
1. Gives correct answer: \$94. Candidates may solve this in various ways, possibly including solution of the simultaneous equations $5x + 4y = 122$ and $21x + 20y = 570$ to give $x = 10$ and $y = 18$ Shows that these are solutions	5	
	5	
		10
Total Points		10

He pays \$570 for 21 fence posts and 20 fence panels.
How much does he pay for 4 fence posts and 3 fence panels? $\$40 + \$54 = \$94$ 3

Show how you figured it out.

Figure 2: Scoring rubric and sample response for expressions and equations summative task. This figure illustrates the scoring guide and a sample student response for a summative middle school task addressing the seventh-grade expressions and equations domain (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015).

Although it appears the sample student received three points for the correct response of \$94 (the cost for 4 fence posts and 3 fence panels), it is not clear how the remaining four points were distributed. Mathematics Assessment Project documents (Shell Center for Mathematics Education at the University of Nottingham & University of California, Berkeley, 2015) assert that although the task involves MP2, MP3, MP6 and MP7, “because of the guidance within the task, [it does] so at a comparatively modest level” (Assessment Task Type section, para. 1). What a ‘comparatively modest level’ means, in terms of student engagement in and assessment of each mathematical practice, and how a ‘modest’ level differs from other levels, are not articulated.

Such lack of clarity is not exclusive to resources from the Mathematics Assessment Project. Rather, the vast majority of practice test items and sample mathematics tasks provided by Common Core and aligned college and career ready resources either: (1) fail to indicate which mathematical practices might be addressed through student engagement in practice items and tasks, (2) neglect to articulate how engagement in the mathematical practices might be revealed in students’ written work, or (3) fail to explicate how engagement in the practice standards, or lack thereof, impacts assessment. Documents from the Partnership for Assessment of Readiness for College and Careers (2012, p. 209) assert the mathematical “practices are challenging to learn, to teach, and to assess . . . [and] function differently from the content in both curriculum and instruction”. As such, the mathematical practices and mathematical content “should not be treated equivalently in assessment either” (Partnership for Assessment of Readiness for College and Careers, 2012, p. 209).

The impetus for the rubric-creating activity presented here was authors’ acknowledgement that assessment of the

mathematical practices should be treated different from mathematical content, and that existing practice item and sample task resources provided little support to teachers in actually assessing mathematical habits of mind. Therefore, the objective of the rubric-creating activity was four-fold. In the first place, the activity serves to model mathematics teacher education or professional learning that supports teachers’ (both prospective and practicing) development of robust conceptions of mathematical processes and proficiencies. Secondly, rubric creation, as described here, illustrates a reflective activity that teachers, math coaches, curriculum developers, professional development instructors, and teacher educators can engage in to support their own operationalization of mathematical habits of mind. Thirdly, such rubric construction helps identify existing and potential connections between Common Core mathematical content and practice standards, and between instruction and assessment of mathematical content and practice standards. Lastly, the rubrics provide sample evaluation tools that attempt to clearly illustrate how the mathematical practices can be exhibited and assessed concurrent with mathematical content—tools that, to date, have been absent from college and career ready standards resources.

Although the rubrics can be used to evaluate the specific mathematics standards and tasks described here, the focus of the report will remain on the creation of the rubrics. In particular, the focus will remain on descriptions of the rubric construction activity and the benefits such a reflective endeavor affords in the development of rich, yet practical conceptions of mathematical habits of mind. Such reflection is propitious for supporting mathematics education stakeholders (teachers, teacher educators) in transforming relatively shallow interpretations of mathematical process and proficiencies into more robust conceptions involving significant meaning and substantive thought.

In addition, rather than placing an emphasis on how participating teachers might have transformed their conceptions as a result of their engagement in the rubric-creating activity, the focus of this report is on the design and management of the activity. The rubric-creating activity was designed and managed to increase the likelihood that participating teachers made their conceptions of the Standards for Mathematical Practice explicit and objects of thought; that is, designed and managed in a manner authors identified as a *reflective rubric* activity. Such a focus requires a detailed discussion of the underlying framework and management of the activity.

5. Framework

The rubric-creating activity is grounded in notions of: (1) beginning with the end in mind (Covey, 2004; Wiggins & McTighe, 2005), (2) decentration (Piaget, 1962; Steffe & Thompson, 2000), (3) reflection-in-action (Schön, 1983), and (4) cognitive residue (Rosenbaum (1972; Salomon, Globerson, & Guterman, 1989). The process of ‘beginning with the end in mind,’ as employed here and based on the habit described by Covey (2004), aligns with Wiggins and McTighe’s (2005) notion of ‘backward design.’ According to Wiggins and McTighe (2005, p. 14), backward design “involves thinking a great deal, first, about the specific learning sought, and the evidence of such learnings, before thinking about what we, as the teacher, will do or provide in teaching and learning activities”. In the rubric-creating activity presented here, the focus was on developing an assessment task or tasks teachers believed: (1) integrated mathematical content and the mathematical practices, (2) faithfully assessed students’ development of intended understandings, fluencies, skills, and dispositions; and (3) supported reflection on the types of instruction that might prepare students to be successful on the assessment task(s). As characterized by Wiggins and McTighe (2005, pp. 14-15), “We cannot say *how* to teach for understanding or *which* material and activities to use until we are quite clear about which specific understandings we are after and what such understandings look like in practice”.

According to Piaget (1962, para. 25), “[D]ecentering . . . i.e., shifting one’s focus and comparing one action with other possible ones, particularly with the actions of other people, leads to an awareness of ‘how’ and to true operations”. As employed in the rubric-creating activity, decentering—“the attempt to imagine one’s experience from another perspective” (Steffe & Thompson, 2000, p. 196)—required those creating the rubric to attempt to see mathematical understandings, reasoning, and ways of thinking from (idealized or epistemic) students’ perspectives. In addition to decentration, Schön’s (1983) notion of ‘reflecting-in-action’—thinking about or

reflecting on an action while engaged in the action, rather than reflecting or thinking back on the action once it has occurred—was central to the design and implementation of the rubric-creating activity. According to Schön’s (1983, p. 68), “When someone reflects-in-action, he becomes a researcher in the practice context . . . [and] does not separate thinking from doing”.

Courtney (2017) detailed practicing secondary school mathematics teachers’ attempts to reconstruct a lesson, as instructional designers, immediately after engaging in the lesson as students of mathematics. According to Courtney (2017, pp. 18-19), teachers lacked meanings (understandings) of sufficient robustness “to sustain productive reflection”, resulting in teachers simply recalling “the order of the non-cognitive actions they engaged in as students of the lesson”. Therefore, the rubric-creating activity attempted to motivate participants to reflect-in-action (*as* they engaged with a task), and, specifically, to focus on reflective participation that developed significant mathematical understandings, reasoning, and ways of thinking.

Finally, as described by Rosenbaum (1972, p. 471), “Cognitive residues are structures that are absorbed or displaced in development”. Furthermore, “Residues become residues because they are not [expressed] . . . [and] are displaced or absorbed by new structures which *are* [expressed] and which, consequently, can become predominant in the cognitive framework” (Rosenbaum, 1972, p. 475). Whereas Rosenbaum (1972) focused on relationships involving cognitive residues and fantasy, Salomon and his colleagues (Salomon, Globerson, & Guterman, 1989) conducted several studies focused on the cognitive effects involving student interactions with technology. Salomon, Perkins, and Globerson (1991, p. 2) distinguish between “two kinds of cognitive effects: Effects *with* technology obtained during intellectual partnership with it, and effects *of* it in terms of the transferable cognitive residue that this partnership leave behind in the form of better mastery of skills and strategies”. Using Salomon, Perkins, and Globerson’s (1991, p. 3) terminology, the rubric-creating activity was designed to focus on “lasting changes in [teachers’] general cognitive capacities *in consequence* of interactions with [the activity]”; that is, on the effects *of* the activity.

6. Methods

Regarding content, the rubrics were designed to assess secondary school students’ performances on assessment tasks addressing the Common Core’s Algebra conceptual category and Reasoning with Equations and Inequalities domain (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 65).

7. Secondary School Algebra Assessment Tasks

The secondary school Algebra assessment tasks and accompanying rubrics were developed with a small number ($n = 12$) of prospective and practicing secondary mathematics teachers (teaching students ages 14–18 years) as part of a graduate level mathematics education course at a mid-size Midwestern university. As an out-of-class assignment, teachers were asked to determine which content standard or standards from the “solve equations and inequalities in one variable” cluster teachers believed could be most effectively assessed (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 65).

The “solve equations and inequalities in one variable” cluster was chosen for three reasons: (1) it is identified as a ‘priority cluster’ on the Smarter Balanced Assessment Consortium’s (2018, p. 16) secondary school *Mathematics Summative Assessment Blueprint*, (2) it is identified as ‘Major Content’ on the Partnership for the Assessment of Readiness for College and Careers’ (2017b, p. 43, 67) *Algebra 1* and *Mathematics I*, and *Mathematics II Summative Assessments*, and (3) participating teachers self-identified strong familiarity with the content. In addition, it was expected that in making their selections, teachers would consider the nature and robustness of their own conceptions of each of the three standards within the cluster. By majority rule, teachers selected the “solve linear equations and inequalities in one variable, including equations with coefficients represented by letters” content standard to focus on as a class (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 65).

Teachers were then requested to work in pairs to create, select, or modify a task or tasks they believed would be most effective at evaluating students’ abilities to engage

in and exhibit proficiency with the “solve linear equations and inequalities in one variable, including equations with coefficients represented by letters” content standard and those mathematical practices teachers deemed most relevant to their chosen task(s). From this collection of tasks, participating teachers and the course instructor (first author) determined which task(s) they believed would be most efficient at measuring students’ abilities to engage productively in the content standard and relevant mathematical practices.

Assessment task for content standard. The assessment task, whose rubrics and their development are the focus of this report, is one of two tasks participating teachers and the first author developed to thoroughly assess, from their point of view, the “solve linear equations and inequalities in one variable, including equations with coefficients represented by letters” content standard and associated mathematical practices. Assessment Task #1 (Figure 3) involves linear equations in one variable and is a modification of a problem one teacher pair found at the IXL Learning (a grade K-12 educational technology company) website. Assessment Task #2 (see Appendix A) involves linear inequalities in one variable and is a modification of a problem a second pair of teachers found at the Illustrative Mathematics website.

Participating teachers decided each task required some modifications—for clarity, in terms of their own and epistemic Algebra 1 students’ understandings of the task, and to more effectively assess both mathematical content and associated practice standards. For example, teachers determined it was propitious to require students make sense of the context of the problem by providing students with a situation where the \$3 per visit cost failed to divide evenly into the amount of money left after paying for a membership. This is illustrated in Part (c) of Assessment Task #1 (Figure 3).

Assessment Task #1

Lucy can pay \$23 for a membership to the science museum and then go to the museum for just \$C per visit.

- Write a linear equation to represent the total cost for Lucy to purchase a membership and visit the science museum t times.
- Rewrite your equation to show the number of times Lucy can visit the museum if she has \$37. Show all of your mathematical thinking and reasoning.
- Given that each visit costs \$3, how many visits can Lucy make? Show all of your mathematical thinking and reasoning.

Figure 3: Assessment task #1. This figure illustrates the first of two tasks designed to assess students’ abilities to solve linear equations and inequalities in one variable, including equations with coefficients represented by letters (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 65).

Identifying mathematical practices addressed by the task. Participating teachers were asked to identify the mathematical practices they believed students might engage in and potentially exhibit engagement in as they worked to

make sense of and toward a solution to Assessment Task #1 (Figure 3). In addition, Common Core and aligned college and career ready mathematics standards documents link the content standard to specific mathematical practices as given

in Table 3. Whereas the mathematical practices depicted in Table 3 for the Partnership for the Assessment of Readiness for College and Careers' (2017a) *End of Year Algebra I Assessment*, Arizona State Board of Education (2014), Kentucky State

Board of Education (2011), and participating teachers are all directly linked to the content standard, the Smarter Balanced Assessment Consortium (2015) indicated their problems were assessed at the cluster level.

Table 3: Standards for Mathematical Practice Connected to Content Standard

Source	Mathematical Practice(s)
Participating Teachers (Graduate Math Methods Course, n = 12)	MP1 - Make sense of problems and persevere in solving them MP2 - Reason abstractly and quantitatively MP4 - Model with mathematics
Arizona Department of Education (2014)	MP2 - Reason abstractly and quantitatively MP7 - Look for and make use of structure MP8 - Look for and express regularity in repeated reasoning
Kentucky Department of Education (2011)	MP2 - Reason abstractly and quantitatively MP6 - Attend to precision MP7 - Look for and make use of structure
Partnership for the Assessment of Readiness for College and Careers (2017a)	MP7 - Look for and make use of structure
Smarter Balanced Assessment Consortium (2015)	MP1 - Make sense of problems and persevere in solving them MP2 - Reason abstractly and quantitatively MP4 - Model with mathematics MP5 - Use appropriate tools strategically MP7 - Look for and make use of structure

As illustrated in Table 3, there is a reasonable degree of variability in which and the number of mathematical practices identified as being connected to the content standard among the various sources, regardless of the task chosen. Furthermore, there was not one mathematical practice unanimously identified as being addressed.

8. Evaluation Criteria for the Task

Brookhart defined rubrics as “descriptive rating scales that are particularly useful for scoring when judgment about the quality of an answer is required” (1999, p. 46). As such, rubrics serve to guide evaluation of the products or processes of students’ thinking and reasoning (Moskal, 2000). Developing criteria that assess mathematical content and the practices concurrently requires more than classifying student responses as ‘minimal,’ ‘partial,’ ‘satisfactory,’ or ‘extended’ without making explicit how each of these classifications differ in what demonstrating such engagement means in the context of working on the task. Rather, evaluation criteria must provide a clear “description of student performance that is required to reach that particular level” (Brahier, 2013, p. 26) in terms of conceptual understanding, procedural skill and fluency,

applications and the “habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing” (Conference Board of the Mathematical Sciences, 2012, p. 19).

The mathematical practices identified in the rubrics presented here derived from participating teachers’ attempts to make sense of and work toward a solution to the tasks (Figure 3, Appendix A), and from this group’s combined images for how epistemic students might engage with, perhaps struggle with, and work toward a solution—albeit not necessarily a correct solution. As indicated previously, due to limitations of space, only the rubrics created to evaluate Assessment Task #1 (Figure 3) will be presented here.

9. Rubric-Creating Activity

Rubric development occurred during structured in-class and online collaborative editing sessions using the Google Docs web application. Throughout several in-class and synchronous online Google Docs sessions, Assessment Task #1 and its accompanying rubrics were used as didactic objects (Thompson, 2013). Thompson (2013) described didactic objects as “displays, diagrams, graphs, mathematical

expressions, or class activities . . . designed conscientiously to support specific reflective conversations” (p. 77). It is important to stress the course instructor purposely managed the rubric-creating activity so that teachers focused on their own and epistemic students’ engagements with the mathematical practices *as* they strove to make sense of and work toward a solution to the task. Such a focus appeared not to be a natural way for participating teachers to engage with a task. Rather, teachers attempted to first solve the task, then consider potential mathematical practices students might engage in; and finally, albeit to a minimal degree, how students might exhibit such engagement.

Prior course experiences with participating teachers highlighted their general lack of attention to the meanings and reasoning they employed *as* they engaged with tasks, during both group and individual activities. Furthermore, teachers were generally constrained to show much in the way

of written work, particularly as it related to articulating their thinking and reasoning. As described by Jonassen and Strobel (2006), there is “cognitive residue evidenced in the artifacts that learners produce . . . that is, when students produce artifact, . . . there is extensive evidence of their thinking in the products” (p. 7). Therefore, the limited recorded (or scripted) artifacts and cognitive residue expressed by teachers appeared to constrain their capacities to reflect on—once they arrived at a solution—the mathematical practices they had engaged in and their students might engage in and potentially reveal. These observations align with results described earlier and characterized by Courtney (2017).

Rubric for part (a) of assessment task #1. The rubric created for Part (a) of Assessment Task #1 is illustrated in Table 4. Although not indicated in the rubric, students receive no points for making no attempt to solve the task.

Table 4: Rubric for Part (a) of Assessment Task #1

4 points	3 points	2 points	1 point
<p>Student writes one of the following:</p> <p>1) Total Cost = $23 + Ct$ or Total Cost = $Ct + 23$</p> <p>2) Cost = $23 + Ct$ or Cost = $Ct + 23$</p> <ul style="list-style-type: none"> • Student creates a coherent representation of the task at hand by abstracting the given situation and representing it symbolically (MP2) • Student considers the meaning of quantities and implicitly addresses the units of ‘C,’ ‘t,’ ‘23,’ the product ‘Ct,’ and the sums ‘Ct + 23,’ or ‘23 + Ct’ (MP2) • Student clearly defines the meaning of the expression ‘23 + Ct’ or ‘Ct + 23’ as ‘Total Cost’ or ‘Cost’ (MP6) • Student takes care to use symbols ‘C’ and ‘t’ (both given in task) to develop their expression (MP6) 	<p>Student writes one of the following:</p> <p>1) Total Cost = $23 + CV$ or Total Cost = $CV + 23$</p> <p>2) Cost = $23 + CV$ or Cost = $CV + 23$</p> <p>In all cases above, student uses ‘incorrect’ symbol to represent the number of museum visits (the task indicates ‘t’ should be used) – student might use ‘V’ or ‘N’ or some other letter to represent the number of museum visits.</p> <p>Student may or may not define the variable used to represent the number of museum visits.</p> <p>a) Student fails to define symbol used to represent number of museum visits (other than ‘t’).</p> <ul style="list-style-type: none"> • Student does not take care to use symbol (‘t’) provided in task to represent number of museum visits (MP6) • Student clearly defines the meaning of the expression ‘23 + CV’ or ‘CV + 23’ as ‘Total Cost’ or ‘Cost,’ (MP6), but does not state the meaning of the symbol ‘V’ (MP6) 	<p>Student writes one of the following:</p> <p>1) $TC = 23 + CN$ or $TC = CN + 23$</p> <p>In case above, student does not define ‘TC’ (implicitly representing the ‘Total Cost’) and does not distinguish between the ‘C’ in ‘TC’ and the ‘C’ representing the cost per visit.</p> <p>1) $C = 23 + CN$ or $C = CN + 23$</p> <p>In case above, student does not define ‘C’ (implicitly representing ‘Total Cost’) and does not distinguish it from the ‘C’ representing the cost per visit.</p> <ul style="list-style-type: none"> • Student might use some other abbreviation for ‘Total Cost’ or ‘Cost,’ but does not make the meaning of the constructed expression, ‘23 + CN’ or ‘CN + 23,’ explicit (MP6) <p>In all cases above, student uses ‘incorrect’ symbol to represent the number of museum visits (the task indicated ‘t’ should be used) – student might use ‘V’ or ‘N’ or some other letter to represent the number of museum visits.</p>	<p>Student writes one of the following:</p> <p>1) Total Cost = $23Ct$</p> <p>2) Total Cost = $C + 23t$</p> <p>3) Cost = $23Ct$</p> <p>4) Cost = $C + 23t$</p> <ul style="list-style-type: none"> • Student does not demonstrate a coherent representation of the task at hand by attending to the meaning of quantities (MP2) • Student is unable to abstract a given situation and represent it symbolically (MP2) • Student does not attend to the meanings and units of the product of ‘23Ct,’ the product ‘23t,’ or the sums ‘C + 23t’ or ‘23t + C’ (MP2) • Student clearly defines the meaning of the expression ‘23Ct,’ ‘C + 23t’ or ‘23t + C’ as ‘Total Cost’ or ‘Cost’ (MP6) <p>5) Total Cost = $23CN$</p> <p>6) Total Cost = $C + 23N$</p> <p>7) Cost = $23CN$</p> <p>8) Cost = $C + 23N$</p> <p>In cases (5) - (8) above, it is assumed student uses ‘incorrect’ symbol to represent the number of museum visits (the task</p>

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- Student creates a coherent representation of the task at hand by abstracting the given situation and representing it symbolically (MP2)
 - Student's representation does not make clear the meaning of 'V,' but representation implicitly identifies quantity as number of museum visits (MP2)
 - Student's representation implicitly attends to the meanings and units of 'C,' '23,' 'V,' the product 'CV' and the sums 'CV + 23,' or '23 + CV' (MP2)
- b) Student defines symbol used to represent number of museum visits (other than 't').
- Student states the meaning of the symbol 'V' or other variable used to represent number of museum visits (MP6)
 - Student clearly defines the meaning of the expression '23 + CV' or 'C + 23V' as 'Total Cost' or 'Cost' (MP6)
 - Student creates a coherent representation of the task at hand by abstracting the given situation and representing it symbolically (MP2)
 - Student makes clear the meaning of 'V,' and implicitly attends to the meanings and units of 'C,' 'V,' '23,' the product 'CV,' and the sums 'CV + 23' or '23 + CV' (MP2)
- Note:** If appropriate, student may receive an additional one-half (1/2) of a point.
- 3) $23 + Ct$ or $Ct + 23$
- Student does not make explicit what constructed expression, '23 + Ct' or 'Ct + 23,' represents (MP6)
- Student may or may not define the variable used to represent the number of museum visits.
- a) Student fails to define symbol used to represent number of museum visits (other than 't').
- Student does not state the meaning of the symbol 'N' or other variable used to represent number of museum visits (MP6)
 - Student does not take care to use symbol ('t') provided in task to represent number of museum visits (MP6)
 - Student's representation does not make clear the meaning of 'N,' but representation implicitly identifies quantity as number of museum visits (MP2), which, in turn, indicates student attended to meanings and units of 'C,' 'N,' '23,' the product 'CN,' and the sums 'CN + 23' or '23 + CN' (MP2)
- b) Student defines symbol used to represent number of museum visits (other than 't')
- Student states the meaning of the symbol 'N' or other variable used to represent number of museum visits (MP6)
 - Student does not take care to use symbol ('t') provided in task to represent number of museum visits (MP6)
 - Student creates a coherent representation of the task at hand (MP2)
 - Student makes clear the meaning of 'V,' and implicitly attends to the meanings units of 'C,' 'N,' '23' the product 'CN,' and the sums 'CN + 23' or '23 + CN' (MP2)
- Note:** If appropriate, student may receive an additional one-half (1/2) of a point.
- indicated 't' should be used) – student might use 'N' or 'V' or some other letter to represent the number of museum visits.
- Student does not demonstrate a coherent representation of the task at hand by attending to the meaning of quantities (MP2)
 - Student is unable to abstract a given situation and represent it symbolically (MP2)
 - Student does not attend to meanings and units of the product of '23CN,' the product '23N,' or the sums 'C + 23N' or '23N + C' (MP2)
 - Student clearly defines the meaning of the expression '23CN,' 'C + 23N' or '23N + C' as 'Total Cost' or 'Cost' (MP6)
 - Student does not take care to use symbol ('t') provided in task to represent number of museum visits (MP6)
- Note:** If student fails to define the symbol used to represent the number of museum visits (other than 't'), then student **may have an additional** one-half (1/2) point **deducted**.
- 9) $TC = 23Ct$
- 10) $TC = C + 23t$ or $TC = 23t + C$
- In cases (9) - (10) above, student does not define 'TC' or distinguish between the 'C' in 'TC' and the 'C' representing the cost per visit.
- 11) $C = 23Ct$
- 12) $C = 23 + Ct$ or $C = Ct + 23$
- In cases (11) - (12) above, student does not define 'C' or distinguish it from the 'C' representing the cost per visit.
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- Student creates a coherent representation of the task at hand by abstracting the given situation and representing it symbolically (**MP2**)
- Student's representation implicitly addresses the meanings and units of 'C,' 't,' '23,' the product 'Ct,' and the sums 'Ct + 23' or '23 + Ct' (**MP2**)

$$4) TC = 23 + Ct \text{ or } 23 + Ct = TC$$

In case above, student does not define 'TC' (implicitly representing the 'Total Cost') or distinguish between the 'C' in 'TC' and the 'C' representing the cost per visit.

$$5) C = 23 + Ct \text{ or } Ct + 23 = C$$

In case above, student does not define 'C' (implicitly representing the 'Total Cost') or distinguish it from the 'C' representing the cost per visit.

- Student might use some other abbreviation for 'Total Cost' or 'Cost,' but does not make their meaning explicit (**MP6**)
- Student creates a coherent representation of the task at hand by abstracting the given situation and representing it symbolically (**MP2**)
- Student's representation implicitly addresses the meanings and units of 'C,' 't,' '23,' the product 'Ct,' and the sums 'Ct + 23' or '23 + Ct' (**MP2**)

- Student might use some other abbreviation for 'Total Cost' or 'Cost,' but does not make their meaning explicit (**MP6**)

For cases (9) - (12) above:

- Student does not demonstrate a coherent representation of the task at hand by attending to the meaning of quantities (**MP2**)
- Student is unable to abstract a given situation and represent it symbolically (**MP2**)
- Student does not attend to the meanings and units of the product of '23Ct,' the product '23t,' or the sums 'C + 23t' or '23t + C' (**MP2**)
- Student takes care to use appropriate symbols (as given in task) to develop expression (MP6), but does not make explicit what constructed expression, '23Ct' or the sums 'Ct + 23' or '23 + Ct' represent (**MP6**)

Student might also use 'incorrect' symbol to represent the number of museum visits (the task indicated 't' should be used) – student might use 'V' or 'N' or some other letter to represent the number of museum visits.

Note: If student does not indicate the meaning of this symbol (other than 't'), then student may have an additional one-half (1/2) point deducted.

$$13) 23Ct$$

$$14) 23 + Ct \text{ or } Ct + 23$$

- Student does not demonstrate a coherent representation of the task at hand by attending to the meaning of quantities (**MP2**)

- Student is unable to abstract a given situation and represent it symbolically (MP2)
- Student does not consider the units of the product of '23Ct,' the product '23t,' or the sums 'C + 23t' or '23t + C' (MP2)
- Student takes care to use appropriate symbols (as given in task) to develop expression (MP6), but does not make explicit what constructed expression, '23Ct' or the sums 'Ct + 23' or '23 + Ct' represent (MP6)

Student might also use 'incorrect' symbol to represent the number of museum visits (the task indicated 't' should be used) – student might use 'V' or 'N' or some other letter to represent the number of museum visits.

Note: If student does not indicate the meaning of this variable (other than 't'), then student **may have an additional one-half (1/2) point deducted** from their score.

The term 'implicitly' is used at various points in the rubric. For example, in the 4-point response (Table 4), the rubric asserts, in reference to MP2 (Reason abstractly and quantitatively), student's representation "implicitly addresses the units of 'C,' 't,' '23,' the product 'Ct,' and the sums 'Ct + 23,' or '23 + Ct.'" The use of such language causes significant pause, considering authors' intent to make explicit how students might and might not demonstrate engagement with specific mathematical practices.

The Partnership for the Assessment of Readiness for College and Careers (2012) asserted its assessments employ 'practice-forward tasks,' defined as tasks in which it is "unlikely or impossible to earn full credit . . . without engaging in the practice" (p. 259). Although the authors believe Assessment Task #1 aligns with this description, participating teachers' and the first author's intent was to make student engagement in the practice(s) *directly observable* in student work. Such observable evidence must walk a thin line between requiring an unrealistic, and perhaps absurd, amount of student work and allowing someone using the rubric to assume students implicitly thought or reasoned in a particular way with

little, or even any, written indication students actually did so. Throughout the development of the rubrics, the question of what, in terms of student written work (the products or artifacts), comprised sufficient evidence of having engaged in a particular mathematical practice or practices were occasions of lengthy and lively discourse among participating teachers and the first author. Therefore, the rubric-creating activity provided a space that supported teachers (and the course instructor, a mathematics teacher educator) in making decisions about what makes sense in terms of required student written work.

At some point in students' mathematics development it would make sense to require they clearly articulate (verbally and in written work) how they attended to the meaning of quantities involved in a task (i.e., MP2). Such occasions, again referring to the 4-point response in the rubric (Table 4), would require student representations to explicitly address the units of complex quantities and their constituent components: '\$C/visit,' 't visits,' 'Ct dollars,' '\$23,' and the sum '(Ct + 23) dollars.' For Assessment Task #1 (Figure 3), it is assumed epistemic Algebra I or Mathematics I students

have moved beyond the need to require such explication. Rather, students would have necessarily considered the meanings and units of such quantities as a habit of mind. Therefore, if instruction holds students accountable for thinking and reasoning in this manner as a classroom norm, then it should be self-evident that over time such thinking and reasoning would habitually occur.

An argument could be made that if students were consistently held accountable for clearly articulating the meaning of quantities (MP2), then they would naturally avoid identifying the total cost of the museum visits ambiguously as ‘CT,’ ‘C,’ or some other abbreviation without indicating such. The authors agree with this line of reasoning and assert this is precisely why students would have one point deducted from their score due to students’ neglect in making such a specification (possible differences between 4-point through 1-point responses)—the mathematical practice would not have yet developed into a habit (Table 4).

Reflective activity. What participating teachers engaged in throughout development of the rubric for Part (a) of Assessment Task #1, and what the authors suspect assisted in the group’s development of all three rubrics and mathematical practice conceptions, is what Glasersfeld (1995) characterized as reflection; the capacity to “step out of the stream of direct experience, to represent a chunk of it, and to look at it as though it [was] direct experience, while remaining aware of the fact that it is not” (p. 90). Such reflection, or reflection-in-action (Schön, 1983), motivated

the group, through the course instructor’s purposeful management of the discussions—both in-class and during synchronous online sessions—to decenter and consider how *students might engage* with the task by reflecting on how *they were engaging* with the task.

Such purposeful actions required the course instructor to take periodic *pedagogical timeouts*, where the group’s focus shifted from their own engagement with the task as mathematics problem-solvers to imagining epistemic students engaging with the task. This shift was not a modest action taken by the course instructor. Rather, the action promoted a significant and considerably different way of thinking about the development of rubrics that assess students’ processes and proficiencies than commonly employed by participating teachers or indicated in existing college and career ready standards resources. Next, participating teachers moved to Part (b) of the task, continuing to take pedagogical timeouts as needed. Such occurrences arose whenever the course instructor noted teachers moving through the rubric-creating process without attempting to decenter.

Rubric for part (b) of assessment task #1. The rubric created for Part (b) of Assessment Task #1 is illustrated in Table 5. As with part (a) of the task, students receive no points for making no attempt to solve the task. Throughout the creation of the rubric for Part (b), teachers modified the rubric for Part (a) as they reflected on misconceptions or issues epistemic students might encounter as students attempted to re-write the linear equation (Table 5).

Table 5: Rubric for Part (b) of Assessment Task #1

4 points	3 points	2 points	1 point
<p>Student shows the following:</p> $37 = 23 + Ct$ $37 - 23 = 23 - 23 + Ct$ $14 = Ct$ $\frac{14}{C} = \frac{Ct}{C}$ $\frac{14}{C} = t$ <p>It is also acceptable for students to state in words that they ‘subtract 23 from both sides’ and ‘divide both sides by C.’ This would still address the request to ‘show all of your mathematical thinking and reasoning.’</p>	<p>Student shows the following:</p> $37 = 23 + Ct$ $14 = Ct$ $\frac{14}{C} = t$ <p>If student used a different symbol to stand for the number of museum visits (such as ‘V’ or ‘N’) in Part (a), defined this variable in Parts (a) or (b), and fails to show their work as above, they may still earn 3 points for Part (b).</p>	<p>Student shows the following:</p> <ol style="list-style-type: none"> $37 = 23Ct$ $\frac{37}{23C} = \frac{23Ct}{23C}$ $\frac{37}{23C} = t$ $37 = C + 23t$ $37 - C = C - C + 23t$ $37 - C = 23t$ $\frac{37 - C}{23} = \frac{23t}{23}$ $\frac{37 - C}{23} = t$ <p>It is also acceptable for students to state in words that they ‘subtract C from both sides’ or ‘divide both sides by 23C.’ This would still address the request to ‘show all of your mathematical thinking and reasoning.’</p>	<p>Student shows the following:</p> <ol style="list-style-type: none"> $t = \frac{37}{23C}$ $t = \frac{37 - C}{27}$ <p>If student used a different symbol to stand for the number of museum visits (such as ‘V’ or ‘N’) in Part (a), defined this variable in Parts (a) or (b), and fails to show their work as above, they may still earn 1 point for Part (b).</p> <ul style="list-style-type: none"> Student makes some sense of quantities involved, but does not demonstrate a coherent representation of the problem at hand by attending to the meaning of quantities; in particular, student does not attend to the meanings and units of $\frac{37}{23C}$, or $\frac{37 - C}{23}$.

If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N') in Part (a), defined this variable in Parts (a) or (b), and shows their work as above, they **may still earn 4 points** for Part (b).

- Student makes sense of quantities and their relationships in problem situations by knowing where the \$37 fits in the equation (MP2)
- Student shows all of their thinking and reasoning, although they might state their algebraic steps in words (MP6)
- In solving the equation for 't,' student can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects; in particular, student sees '23 + Ct' as the total cost, but also as being made up of the component parts '\$23' and '\$(Ct),' which in turn, is made up of '\$C/visit' and 't visits'; such a structural view is necessary to solve for t (MP7)

Note: If student shows all of their work (as in above) but makes a calculation error (e.g., $37 - 23 = 12$), they **may have an additional** one-half (1/2) point **deducted** from their score.

Note: If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N'), failed to define this symbol in Part (a) or (b), but shows their work as above, they **may have** one-half (1/2) point **deducted** from their score.

- Student makes sense of quantities and their relationships in problem situations by knowing where the \$37 fits in the equation (MP2)
- Student **fails to explicitly show** all of their thinking and reasoning (student **does not** make their algebraic thinking explicit in solving for 't') (MP6)
- In solving the equation for 't,' student can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects; in particular, student sees '23 + Ct' as the total cost, but also as being made up of the component parts '\$23' and '\$(Ct),' which in turn, is made up of '\$C/visit' and 't visits'; such a structural view is necessary to solve for t (MP7)

Note: If student shows all of their work (as in above) but makes a calculation error (e.g., $37 - 23 = 12$), they **may have an additional** one-half (1/2) point **deducted** from their score.

Note: If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N'), failed to define this variable in Part (a) or (b), and fails to show their work as above, they **may have an additional** one-half (1/2) point **deducted** from their score.

If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N') in Part (a), defined this variable in Parts (a) or (b), and shows their work as above, they **may still earn 2 points** for Part (b).

- Student makes some sense of quantities involved, but **does not** demonstrate a coherent representation of the problem at hand by attending to the meaning of quantities; in particular, student does not attend to the meanings or units of ' $\frac{37}{23C}$ ', or ' $\frac{37-C}{23}$ ', and the incoherence of either to represent the number of museum visits (MP2)
- Student makes sense of the quantity \$37 and its relationship in the problem by knowing where it fits in the equation (MP2)
- Student shows all of their thinking and reasoning, although they might state their algebraic steps in words (MP6)
- In solving the equation for 't,' student can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects; in particular, student sees '23Ct' as the total cost, but also as being made up of the component parts '23' and '(Ct),' which in turn, is made up of 'C' and 't'; such a structural view is necessary to solve for t (MP7), but student's inattention to the meanings and units of these component parts (MP2) **constrains** them from taking note of the incoherence of their solution.

and the incoherence of either to represent the number of museum visits (MP2)

- Student fails to explicitly show all of their thinking and reasoning (student does not make their algebraic thinking explicit in solving for 't') (MP6)
- Student makes sense of the quantity \$37 and its relationship in the problem by knowing where it fits in the equation (MP2)
- In solving the equation for 't,' student can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects; in particular, student sees '23Ct' as the total cost, but also as being made up of the component parts '23' and '(Ct),' which in turn, is made up of 'C' and 't'; such a structural view is necessary to solve for t (MP7), but student's inattention to the units of these component parts (MP2) **constrains** them from identifying the incoherence of their solution

Note: If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N'), failed to define this variable in Parts (a) or (b), and fails to show their work as above, they may have an additional one-half (1/2) point deducted from their score.

3) Student substitutes the \$37 incorrectly into the equation, but is still able to solve (albeit incorrectly) for 't':

- Total Cost = $23 + 37t$
- Cost = $37 + 23t$

- In solving the equation for 't,' student can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects; in particular, student sees '23 + 37t' as the total cost, but also as being made up of the component

Note: If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N'), failed to define this variable in Part (a) or (b), but shows their work as above, they may have one-half (1/2) point deducted from their score.

parts '23' and '(37t),' which in turn, is made up of '37' and 't'; such a structural view is necessary to solve for t (**MP7**), but student's inattention to the meanings and units of these component parts (**MP2**) constrains them from taking note of the incoherence of their solution

Note: If student fails to show how solved for 't' either algebraically or in words, then they may have an additional one-half (1/2) point deducted from their score.

- If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N') in Part (a), defined this variable in Parts (a) or (b), incorrectly substitutes the \$37 as above, and show their work in solving for 't,' they may still earn 1 point for Part (b).

Note: If student used a different symbol to stand for the number of museum visits (such as 'V' or 'N'), failed to define this variable in Parts (a) or (b), incorrectly substitutes the \$37 as above, and does not show their work in solving for 't,' they may have one-half (1/2) point deducted from their score.

- 4) Student substitutes the \$37 incorrectly into the equation, and is unable to solve for 't':
- a) $\text{Cost} = 23 + C(37)$
 - b) $\text{Total Cost} = 23C(37)$

In cases (3) – (4) above:

- Student does not make sense of quantities involved and does not demonstrate a coherent representation of the problem at hand by attending to the meaning and units of quantities; in particular, student incorrectly substitutes the \$37 into the equation, does not attend to the units of the quantities involved, and does not attend to the incoherence of the representation of the problem at hand (**MP2**)

Throughout the development of the rubric for Part (b), the first author continued to manage the activity so that teachers were motivated to articulate and provide

instantiations of their thinking and reasoning, in an attempt to provide teachers with robust cognitive residue with which to reflect. In Part (b) of Task Assessment #1 (Figure 3),

students could receive extra (or bonus) points by placing constraints on ‘C,’ the cost per visit. Specifically, for $C \neq 0$, $t = \frac{14}{C}$ visits; whereas, for $C = 0$, the number of visits (t) is unlimited—although the museum’s hours of operation, Lucy’s availability to visit the museum, among other factors, would all play a role in the maximum number of visits she could make. This further demonstrates students’ abilities to make sense of quantities and their relationships in problem situations (MP2).

In addition, if a student indicated ‘ t ’ must be a whole number, that there cannot be fractional parts of a visit to the museum (a part of a museum visit does not make sense), then teachers viewed the student as interpreting “their mathematical results in the context of the situation and

reflect[ing] on whether the results make sense” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 7), which demonstrates MP4 (Model with mathematics)—a mathematical practice addressed explicitly in Part (c). Finally, the authors envision classroom instruction following Assessment Task #1 to be managed so the constraints on ‘C’ (the cost per museum visit) and ‘ t ’ (the total number of museum visits), as described above, are presented and discussed. Thus, supporting students’ development of increasingly sophisticated mathematical habits of mind.

Rubric for part (c) of assessment task #1. The rubric created for Part (c) of Assessment Task #1 is illustrated in Table 6. Students again receive no points for making no attempt to solve the task.

Table 6: Rubric for Part (c) of Assessment Task #1

4 points	3 points	2 points	1 point
<p>Student shows the following:</p> $t = \frac{14}{3} \text{ museum visits}$ <p>or,</p> $t = 4\frac{2}{3} \text{ museum visits}$ <p>or,</p> $t \approx 4.667 \text{ or } 4.667 \text{ or } 4.7 \text{ museum visits}$ <p>so,</p> $t = 4 \text{ museum visits}$ <p>Student concludes Lucy can visit the museum 4 times.</p> <p>If student used a different symbol to stand for the number of museum visits (such as ‘V’ or ‘N’) in Part (a) and shows their work as above, they may still earn 4 points for Part (c).</p> <p>Note: If students do not take care about specifying units of measure of their solution (number of museum visits), they may have one-half (1/2) point deducted from their score (MP6)</p> <ul style="list-style-type: none"> Student calculates accurately and efficiently, and expresses the numerical answer with a degree of precision appropriate for the problem context (number of museum visits) (MP6) 	<p>Student shows the following:</p> $t = \frac{14}{3} \text{ museum visits}$ <p>or,</p> $t = 4\frac{2}{3} \text{ museum visits}$ <p>or,</p> $t \approx 4.667 \text{ or } 4.667 \text{ or } 4.7 \text{ museum visits}$ <p>Student concludes Lucy can visit the museum $4\frac{2}{3}$ (or 4.67 or 4.667 or 4.7) times.</p> <p>If student used a different symbol to stand for the number of museum visits (such as ‘V’ or ‘N’) in Part (a), and shows their work as above, they may still earn 3 points for Part (c).</p> <p>Note: If students do not take care about specifying units of measure of their solution (number of museum visits), they may have one-half (1/2) point deducted from their score (MP6)</p> <ul style="list-style-type: none"> Student calculates accurately and efficiently, but fails to express the numerical answer with a degree of precision appropriate for the problem context (number of museum visits) (MP6) 	<p>Student writes one of the following:</p> $1) t = \frac{37}{23(3)} = \frac{37}{69} = 0.536$ <p>museum visits</p> <p>Student concludes Lucy can make no visits to the museum, because she cannot physically attend the museum 0.5 of a time (MP4).</p> $2) t = \frac{37-3}{27} = \frac{34}{27} = 1.259$ <p>museum visits</p> <p>Student concludes Lucy can make one visit to the museum, because she cannot physically attend the museum 1.3 of a time (MP4).</p> <p>If student used a different symbol to stand for the number of museum visits (such as ‘V’ or ‘N’) in Part (a) and shows their work as in (1) – (2) above, they may still earn 2 points for Part (c).</p> <p>Note: If students do not take care about specifying units of measure of their solution (number of museum visits), they may have one-half (1/2) point deducted from their score (MP6)</p> <ul style="list-style-type: none"> Student calculates accurately and efficiently, and expresses the numerical answer with a 	<p>Student writes one of the following:</p> $1) t = \frac{37}{23(3)} = \frac{37}{69} = 0.536$ <p>museum visits</p> <p>Student concludes Lucy can make 0.536 or 0.54 or 0.5 visits to the museum.</p> $2) t = \frac{37-3}{27} = \frac{34}{27} = 1.259$ <p>museum visits</p> <p>Student concludes Lucy can make 1.259 or 1.26 or 1.3 visits to the museum.</p> <p>If student used a different symbol to stand for the number of museum visits (such as ‘V’ or ‘N’) in Part (a) and shows their work as in (1) – (2) above, they may still earn 1 point for Part (c).</p> <p>3) Other outcomes in which student has either incorrectly developed expression to model situation and/or incorrectly substitutes into equation for ‘Total Cost’ (the \$37) and/or ‘C’ (the cost per museum visit).</p> <p>Note: If students do not take care about specifying units of measure of their solution (number of museum visits), they may have one-half (1/2) point deducted from their score (MP6)</p>

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| <ul style="list-style-type: none"> • Student proficiently applies mathematics to solve problems arising in the realistic situation of visiting a museum (MP4) • Student interprets their mathematical result to make sense in the context of the given situation (number of museum visits) by rounding to the correct quantity (MP4) | <ul style="list-style-type: none"> • Student proficiently applies mathematics to solve problems arising in the realistic situation of visiting a museum (MP4) • Student fails to interpret their mathematical result to make sense in the context of the given situation (number of museum visits) by rounding to the correct quantity (MP4) | <ul style="list-style-type: none"> • Student proficiently applies mathematics to solve problems arising in the realistic situation of visiting a museum (MP4) • Student does not proficiently apply mathematics to solve problems arising in the realistic situation of visiting a museum (MP4) • Student interprets their mathematical result to make sense in the context of the given situation (number of museum visits) by rounding to the correct quantity (MP4) | <ul style="list-style-type: none"> • Student calculates accurately and efficiently, but does not express the numerical answer with a degree of precision appropriate for the problem context (number of museum visits) (MP6) • Student does not proficiently apply mathematics to solve problems arising in the realistic situation of visiting a museum (MP4). • Student fails to interpret their mathematical result to make sense in the context of the given situation (number of museum visits) by rounding to the correct quantity (MP4) |
|--|--|---|---|
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Throughout the creation of the rubric for Part (c), teachers re-visited and modified the rubrics for Parts (a) and (b). In addition, the course instructor continued to manage the activity, taking pedagogical timeouts as needed, to motivate teachers to articulate and provide instantiations of their own and epistemic students' thinking and reasoning.

As clearly illustrated in the three rubrics (Tables 4, 5, and 6) and asserted by the Partnership for Assessment of Readiness for College and Careers (2017b, p. 13), "Standards for Mathematical Practice interact and overlap with each other, and several may be used together in solving a given problem". In fact, throughout the creation process, it was difficult, at times, for the group to pin down which of two potentially engaged mathematical practices might be at play—or whether both might be equally addressed. Such occasions provided valuable instances for participating teachers to discuss and reflect on how they and epistemic students might exhibit the mathematical practices in the context of making sense of and working toward a solution to the task.

9. Discussion

One point of criticism the authors have received pertaining to the rubrics involves the notion that these rubrics cannot be used by teachers to evaluate student work for tasks other than Assessment Task #1 (Figure 3). Such criticism seems warranted in that each rubric lacks broad descriptions, such as: "The student demonstrated only partial understanding of the mathematical content and practices essential to the task" or "Student response includes 1 of the 3 elements of a full credit response". For these critics, the level of specificity demonstrated here limits each rubric's usefulness to teachers,

teacher educators, and other stakeholders. An additional point of criticism involves the view the three rubrics are not easy to employ; that is, the assertion, "Only those involved in the rubrics' creation would be able to efficiently utilize them."

In response, it must be noted that the rubrics designed to be used to assess student understanding of the "solve linear equations and inequalities in one variable, including equations with coefficients represented by letters" content standard for *any* task was *not* a goal of the rubric-creating activity. Rather, a *secondary* goal of the activity was to develop rubrics that could be used to assess student understanding of the content standard and associated mathematical habits of mind for those tasks participating teachers located from existing resources, modified, or created themselves (Assessment Tasks #1 and #2). The *primary* goal of the activity was to engage teachers in a reflective endeavor that supported teachers' development of rich, yet pragmatic conceptions of mathematical habits of mind, while constructing assessments that authentically integrated mathematical content and the mathematical practices.

After completing the rubrics created to assess Assessment Tasks #1, participating teachers felt quite confident that the rubrics would productively evaluate students' abilities to: (1) "solve linear equations and inequalities in one variable, including equations with coefficients represented by letters" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 65), and (2) engage in and exhibit engagement in mathematical practices MP2 (Reason abstractly and quantitatively), MP4 (Model with mathematics), MP6 (Attend to precision), and MP7 (Look for and make use of structure). In addition, participating teachers asserted the rubric-creating activity

provided them with significant conceptions of the types of engagements that would prepare students to be successful on Assessments Tasks #1 and #2 (Figure 3, Appendix A).

The authors do not claim the assessment tasks or rubrics presented here are unique in their ability to assess the identified mathematical content and practices, nor that the mathematical practices identified as being addressed by the assessment tasks are exhaustive—although after engaging in the rubric-creating activity, authors and participating teachers felt strongly that MP2, MP4, MP6, and MP7 (Table 1) are the mathematical practices most relevant to Assessment Task #1. More important than identifying which mathematical practices are associated with a task, the authors believed the rubrics and characterizations of their constructions provide insight into and promotes discussion about how mathematics educators at all grade levels can authentically assess the integration of the mathematical content and practice standards. As such, the authors believe this report has the potential to initiate productive discourse around not only the design of such integrated assessments, but also faithful assessment of mathematical habits of mind in general. Furthermore, the type of demanding conceptual analysis (Thompson, 2008) on assessment tasks, as presented here, can serve as a productive reflective endeavor that highlights where students might struggle and how instruction might be designed to focus on natural connections between mathematics content and mathematical habits of mind.

Failure to assess the integration of mathematical content and the practice standards, or to authentically assess the practices at all, has the potential to undermine the significance of mathematical habits of mind. As indicated in Common Core documents (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), the mathematical practices change through the grades as students grow in mathematical maturity and in the sophistication with which they apply mathematics. As such, instruction and assessments need to ensure grade- or course-appropriate mathematical practice expectations.

As the rubric-creating activity and rubrics presented here illustrate, developing evaluation criteria that assess the integration of content and the mathematical practices is not a trivial matter; their development can be quite a time-intensive and cognitive process. Supporting teachers' development of robust conceptions of mathematical processes and proficiencies involves significant meaning and substantive thought. Therefore, the authors recommend teachers engage in reflective rubric-creating activities for all 'major content' of their grade(s) or course(s), provided they engage in such activities with colleagues and allow themselves multiple years to develop assessments and accompanying rubrics for an entire curriculum (Achieve the Core, 2018; Partnership for the Assessment of Readiness for College and

Careers, 2017b; Smarter Balanced Assessment Consortium, 2018). Finally, the detailed descriptions of the design and management of the reflective rubric-creating activity presented here have the potential to provide others (teacher educators) with a protocol for constructing and managing similar engagements.

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Appendix A

Assessment Task #2

Scott was given the following inequality to solve:

$$\text{Solve for } x: \frac{5}{18} - \frac{x-2}{9} \leq \frac{x-4}{6}$$

Scott solved the problem showing the following work (Note: "line #" indicates the steps in Scott's problem-solving process):

$$\text{Line 1: } \frac{5}{18} - \frac{x-2}{9} \leq \frac{x-4}{6}$$

$$\text{Line 2: } \frac{5}{18} - \frac{2}{2} \cdot \frac{x-2}{9} \leq \frac{3}{3} \cdot \frac{x-4}{6}$$

$$\text{Line 3: } \frac{5}{18} - \frac{2x-2}{9} \leq \frac{3x-4}{6}$$

$$\text{Line 4: } 5 - (2x - 2) \leq 3x - 4$$

$$\text{Line 5: } 5 - 2x + 2 \leq 3x - 4$$

$$\text{Line 6: } 7 - 2x \leq 3x - 4$$

$$\text{Line 7: } -5x \leq -11$$

$$\text{Line 8: } x \leq \frac{11}{5}$$

- a) There are two lines where mathematical errors occur in Scott's work (see above). Identify at what step each mathematical error occurred and explain why they are mathematically incorrect.

The first mathematical error occurred going from line ____ to line ____.

Why it is incorrect:

The second mathematical error occurred going from line ____ to line ____.

Why it is incorrect:

- b) Solve the inequality correctly.